

# General analysis of decay chains with three-body decays involving missing energy

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In collaboration with Prof. Ayres Freitas,

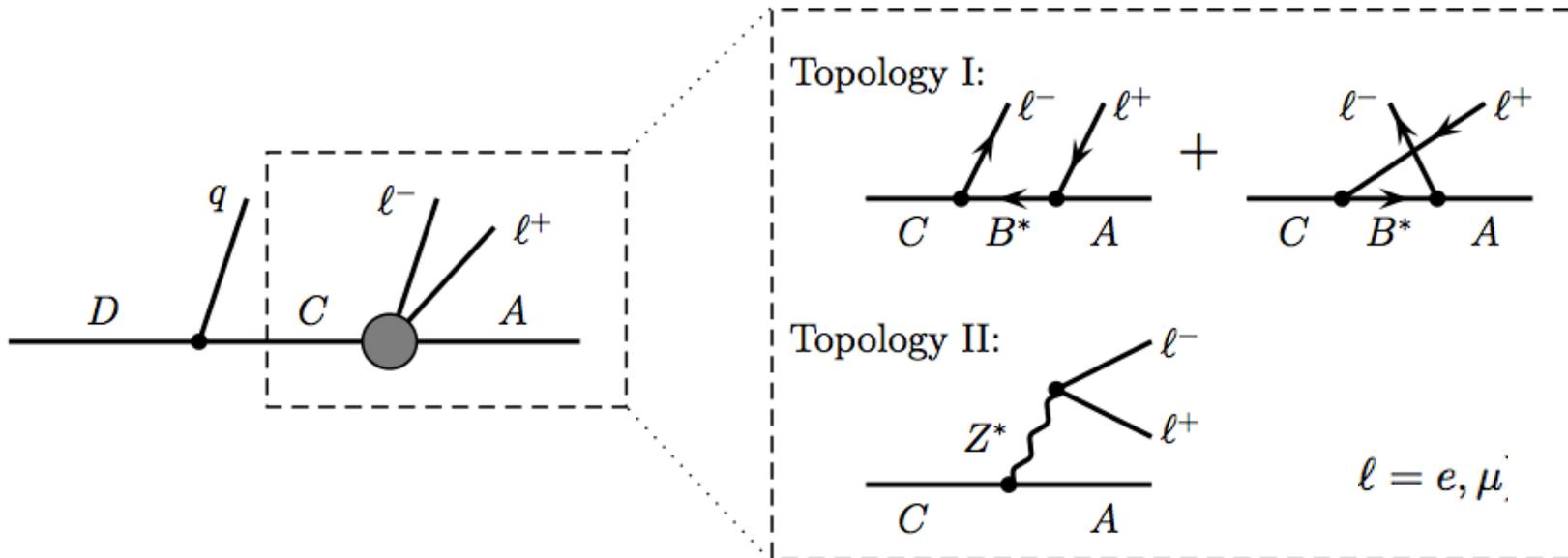
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# Purpose

- Properties of new heavy particles beyond the standard model.  
e.g.
  - Mass
  - Spin
  - Coupling
- Focusing on channels in which the new heavy particles decay into the stable Weakly Interacting Massive Particles (missing energy in the detector or dark matter candidates)
- Model independent approach; making only minimum assumptions
- Consider all possible spin assignments and keep the couplings arbitrary.

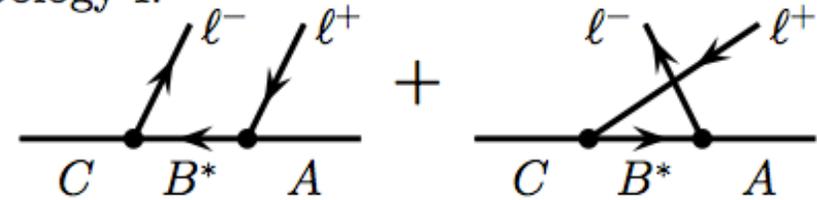
# Topology



- D: Color triplet and charged particle
- C: Neutral self-conjugate massive particle
- B: Charged massive particle
- A: Neutral self-conjugate massive particle; Dark matter candidate
- Z: Standard Model Z boson
- Mass hierarchy :  $M_D > M_C$ ;  $M_B > M_C > M_A$  (for topology I) or  $M_Z > M_C > M_A$  (for topology II) so that particle C decay off-shell to particle B. ( This can often happen in scenarios with relatively small splitting in the mass spectrum of new physics particles. )
  - For particle C decaying on-shell to particle B [JHEP 0810 ,081 (2008) M. Burns et. al.]
  - $M_B \gg M_C$  [JHEP 1008, 035 (2010) L. Edelhaeuser et. al.]

# Spin assignment

Topology I:



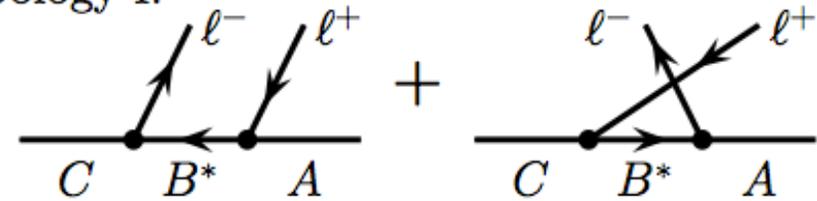
$F$ =Fermion,  $S$ =Scalar,  $V$ =Vector,

Topology I

$S$	$D$	$C$	$B$	$A$
1	S	F	S	F
2	F	S	F	S
3	F	S	F	V
4	F	V	F	S
5	F	V	F	V
6	S	F	V	F

# Spin assignment

Topology I:



F=Fermion, S=Scalar, V=Vector,

e.g.

$$\tilde{q} \rightarrow \tilde{\chi}_2^0 \rightarrow \tilde{\ell}^* \rightarrow \tilde{\chi}_1^0$$

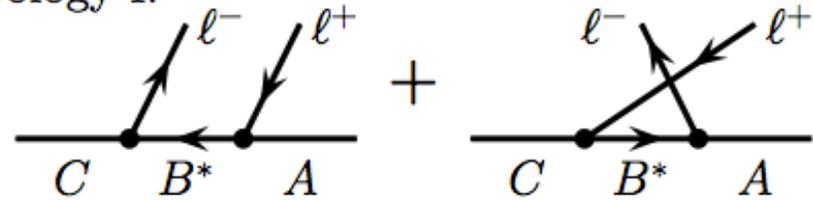
$\tilde{q}$ ,  $\tilde{\ell}$ , and  $\tilde{\chi}_i^0$  are squark, slepton and neutralinos in MSSM.

Topology I

S	D	C	B	A
1	S	F	S	F
2	F	S	F	S
3	F	S	F	V
4	F	V	F	S
5	F	V	F	V
6	S	F	V	F

# Spin assignment

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*F=Fermion, S=Scalar, V=Vector,*

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Topology I

	<i>S</i>	<i>D</i>	<i>C</i>	<i>B</i>	<i>A</i>
1	S	F	S	F	F
2	F	S	F	S	S
3	F	S	F	V	V
4	F	V	F	S	S
5	F	V	F	V	V
6	S	F	V	F	F

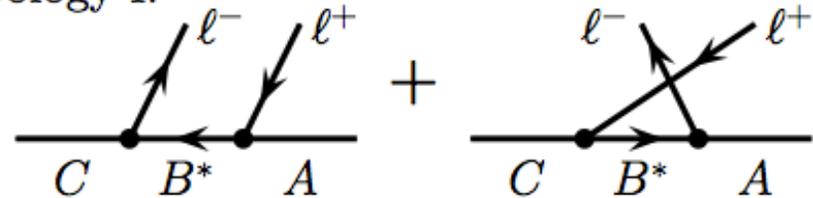
Topology II

7	F	S		S	
8	F	S		V	
9	F	V		S	
10	F	V		V	
11	S	F		F	

# Spin assignment

F=Fermion, S=Scalar, V=Vector

Topology I:



e.g.

$$\tilde{q} \rightarrow \tilde{\chi}_2^0 \rightarrow \tilde{\ell}^* \rightarrow \tilde{\chi}_1^0$$

$\tilde{q}$ ,  $\tilde{\ell}$ , and  $\tilde{\chi}_i^0$  are squark, slepton and neutralinos in MSSM.

Topology I

	S	D	C	B	A
Topology I	1	S	F	S	F
	2	F	S	F	S
	3	F	S	F	V
	4	F	V	F	S
	5	F	V	F	V
	6	S	F	V	F

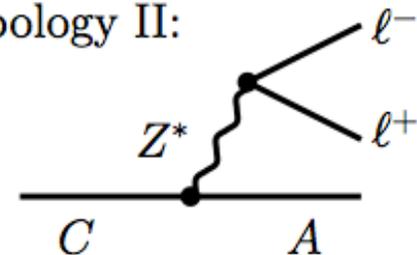
Topology II

Topology II	7	F	S		S
	8	F	S		V
	9	F	V		S
	10	F	V		V
	11	S	F		F

e.g.

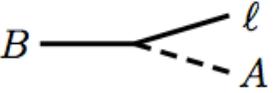
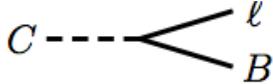
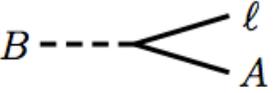
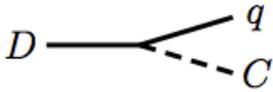
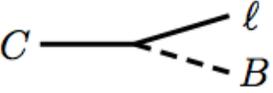
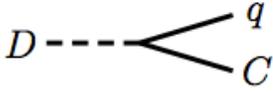
$$\tilde{q} \rightarrow \tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0$$

Topology II:



# Coupling assignment : Topology I

For scalar-fermion-fermion vertices

	$\bar{\psi}_B A (a_L \omega_- + a_R \omega_+) \psi_\ell + \text{h.c.}$		$\bar{\psi}_B C (b_L \omega_- + b_R \omega_+) \psi_\ell + \text{h.c.},$
	$\bar{\psi}_A B (a_L \omega_- + a_R \omega_+) \psi_\ell + \text{h.c.}$		$\bar{\psi}_D C (c_L \omega_- + c_R \omega_+) \psi_q + \text{h.c.}$
	$\bar{\psi}_C B (b_L \omega_- + b_R \omega_+) \psi_\ell + \text{h.c.},$		$\bar{\psi}_C D (c_L \omega_- + c_R \omega_+) \psi_q + \text{h.c.}$

where  $\omega_\pm = \frac{1}{2}(1 \pm \gamma_5)$ .

For vector-fermion-fermion vertices, replace A by  $\cancel{A}$

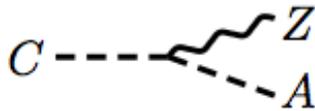
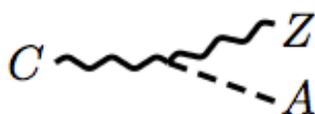
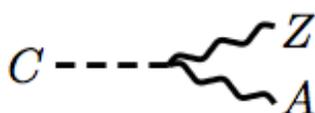
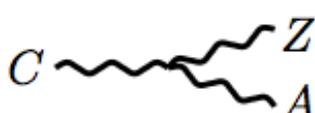
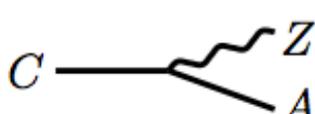
- Only relative sign and magnitude of  $a_L$  and  $a_R$  is known

$$\begin{aligned} a_L &= \cos \alpha, & b_L &= \cos \beta, & c_L &= \cos \gamma, \\ a_R &= \sin \alpha, & b_R &= \sin \beta, & c_R &= \sin \gamma. \end{aligned}$$

$$\alpha \in [-\pi/2, \pi/2], \beta, \gamma \in [0, \pi/2].$$



# Coupling assignment : Topology II

	$i\vec{C}\vec{\partial}_\mu A Z^\mu,$
	$-C_\mu A Z^\mu,$
	$-C A_\mu Z^\mu,$
	$(C_\mu A_\nu - A_\mu C_\nu)\partial^\mu Z^\nu + \text{cycl.},$
	$\bar{\psi}_C \underline{\gamma_\mu \gamma_5} \psi_A Z^\mu,$

- Since the Z-boson is CP-odd, while the self-conjugate A and C are C-even

# Invariant-mass distribution: S=1-6

- Dilepton:

$$\begin{aligned} \frac{1}{\Gamma} \frac{d\Gamma}{d\hat{m}_{\ell\ell}} = & (\cos^2 \alpha \sin^2 \beta + \sin^2 \alpha \cos^2 \beta) f_1^{(\ell\ell)}(\hat{m}_{\ell\ell}^2; m_A^2, m_B^2, m_C^2) \\ & + (\cos^2 \alpha \cos^2 \beta + \sin^2 \alpha \sin^2 \beta) f_2^{(\ell\ell)}(\hat{m}_{\ell\ell}^2; m_A^2, m_B^2, m_C^2) \\ & + (\cos \alpha \sin \alpha \cos \beta \sin \beta) f_3^{(\ell\ell)}(\hat{m}_{\ell\ell}^2; m_A^2, m_B^2, m_C^2), \end{aligned}$$

- The last term comes from the interference term and is sensitive to the sign of the couplings.
- Two fold ambiguity:  $\{\alpha, \beta\} \rightarrow \{\text{sign } \alpha (\frac{\pi}{2} - |\alpha|), \frac{\pi}{2} - \beta\}$

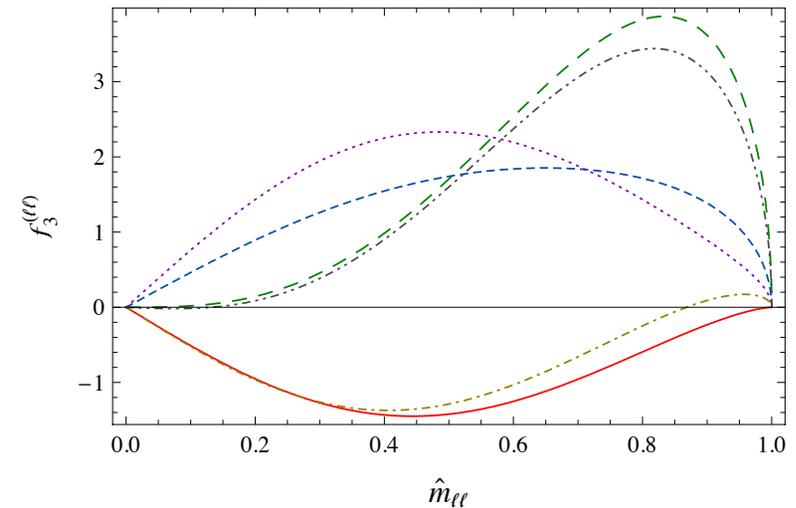
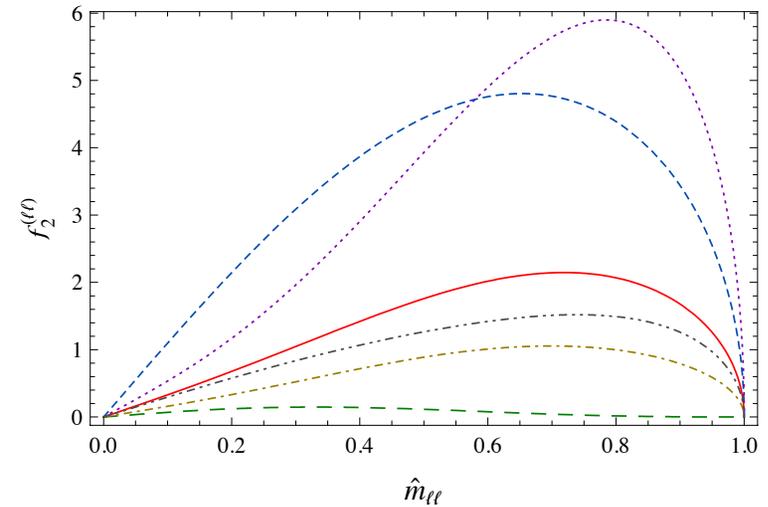
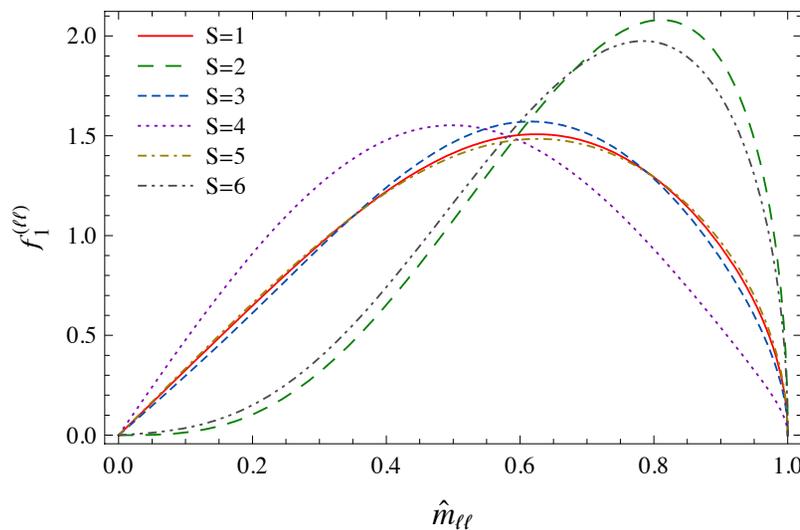
# Invariant-mass distribution: S=1-6

- Dilepton:

MC=184

MB=200

MA=98



- The overall normalization constants have been fixed by requiring that  $f_1^{(\ell\ell)}$  is unit-normalized.
- $f_1^{(\ell\ell)}$  and  $f_2^{(\ell\ell)}$  must be positive because one can choose the couplings such that only one of these terms survives. (since the physical distribution corresponds to a probability distributions)

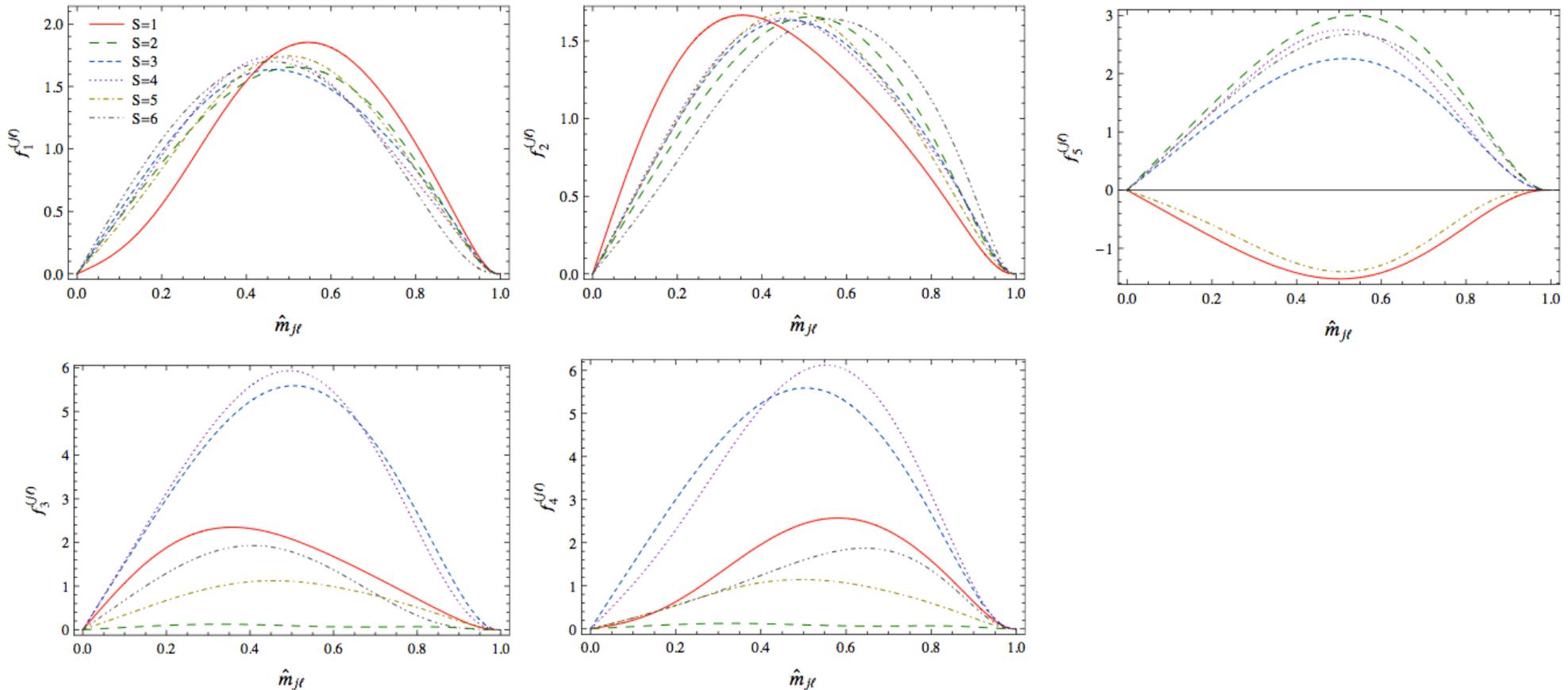
# Invariant-mass distribution: S=1-6

- Jet-lepton:

$$\begin{aligned} \frac{1}{\Gamma} \frac{d\Gamma}{d\hat{m}_{j\ell}} = & (\cos^2 \alpha \sin^2 \beta \cos^2 \tilde{\gamma} + \sin^2 \alpha \cos^2 \beta \sin^2 \tilde{\gamma}) f_1^{(j\ell)}(\hat{m}_{j\ell}^2; m_A^2, m_B^2, m_C^2, m_D^2) \\ & + (\cos^2 \alpha \sin^2 \beta \sin^2 \tilde{\gamma} + \sin^2 \alpha \cos^2 \beta \cos^2 \tilde{\gamma}) f_2^{(j\ell)}(\hat{m}_{j\ell}^2; m_A^2, m_B^2, m_C^2, m_D^2) \\ & + (\cos^2 \alpha \cos^2 \beta \cos^2 \tilde{\gamma} + \sin^2 \alpha \sin^2 \beta \sin^2 \tilde{\gamma}) f_3^{(j\ell)}(\hat{m}_{j\ell}^2; m_A^2, m_B^2, m_C^2, m_D^2) \\ & + (\cos^2 \alpha \cos^2 \beta \sin^2 \tilde{\gamma} + \sin^2 \alpha \sin^2 \beta \cos^2 \tilde{\gamma}) f_4^{(j\ell)}(\hat{m}_{j\ell}^2; m_A^2, m_B^2, m_C^2, m_D^2) \\ & + (\cos \alpha \sin \alpha \cos \beta \sin \beta) f_5^{(j\ell)}(\hat{m}_{j\ell}^2; m_A^2, m_B^2, m_C^2, m_D^2), \end{aligned}$$

- The last term comes from the interference term.
- Two fold ambiguity:  $\{\alpha, \beta, \gamma\} \rightarrow \{\text{sign } \alpha (\frac{\pi}{2} - |\alpha|), \frac{\pi}{2} - \beta, \frac{\pi}{2} - \gamma\}$ .

# Invariant-mass distribution: $S=1-6$

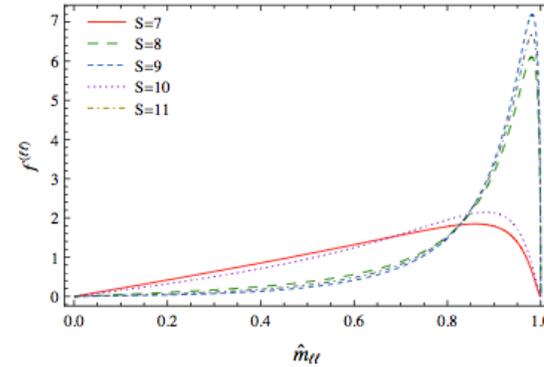


- The overall normalization constants have been fixed by requiring that  $f_1^{(j\ell)}$  is unit-normalized.
- $f_1-f_4$  must be positive because one can choose the couplings such that only one of these terms survives. (since the physical distribution corresponds to a probability distributions)

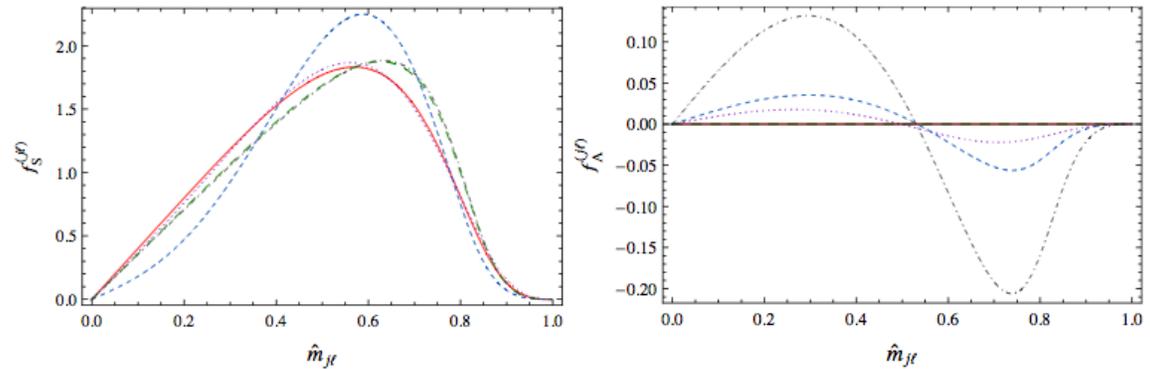
# Invariant-mass distribution: S=7-11

- Dilepton:

MD=565 GeV  
MC=184 GeV  
MA=98 GeV



- Jet-lepton:



$$\frac{1}{\Gamma} \frac{d\Gamma}{d\hat{m}_{\ell\ell}} = f^{(\ell\ell)}(\hat{m}_{\ell\ell}^2; m_A^2, m_B^2, m_C^2),$$

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\hat{m}_{j\ell}} = f_S^{(j\ell)}(\hat{m}_{j\ell}^2; m_A^2, m_B^2, m_C^2, m_D^2) + \cos 2\tilde{\gamma} f_A^{(j\ell)}(\hat{m}_{j\ell}^2; m_A^2, m_B^2, m_C^2, m_D^2).$$

- The overall normalization constants have been fixed by requiring that  $f_S^{(j\ell)}$  is unit-normalized.

# Fitting Procedure

“Data” :  $S = 1, \alpha = 0, \beta = \pi/2, \tilde{\gamma} = 0, m_B = 200 \text{ GeV}$   
(corresponding to the MSSM decay chain  $\tilde{q}_L \rightarrow \tilde{\chi}_2^0 \rightarrow \tilde{l}_L^* \rightarrow \tilde{\chi}_1^0$ );

- Based on 1000 events and 10 bins histograms.
- Perform  $\chi^2$  fit of the theoretical distribution functions to these fake “data” histogram for each of the spin configurations  $S=| - |$ , searching for the minimum value as a function of the parameters  $\alpha, \beta, \tilde{\gamma}$ , and  $m_B$ .

# Results

“Data”, using only  $\hat{m}_{\ell\ell}$  distribution:

$S$	$\min \chi^2$	best-fit parameters		
		$\alpha$	$\beta$	$m_B$ [GeV]
1 [SF $\overline{S}F$ ]	0.00	0.00	1.57	200.0
2 [FSFS]	0.00	-1.22	1.05	209
3 [FSFV]	0.00	+1.14	0.43	197.7
4 [FVFS]	0.27	-1.34	0.23	216
5 [FVFV]	0.05	-0.38	0.38	197
6 [SFVF]	0.05	-0.65	0.92	191.3

$S$	$\min \chi^2$
7 [FSS]	140
8 [FSV]	3100
9 [FVS]	4200
10 [FVV]	290
11 [SFF]	3700

- Difficult to distinguish “data” from  $S=2-6$ .
- There are 3 parameters,  $\alpha$ ,  $\beta$  and  $m_B$ , which can be adjusted so as to mimic the data distribution.
- $S=7-11$  can be distinguished from “data”, because there are no free parameters to adjust.
- Also  $S=7-11$  and “data” come from different topology.

# Results

“Data”, using only  $\hat{m}_{\ell\ell}$  distribution:

$S$	$\min \chi^2$	best-fit parameters		
		$\alpha$	$\beta$	$m_B$ [GeV]
1 [SFSF]	0.00	0.00	1.57	200.0
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6 [SFVF]	0.05	-0.65	0.92	191.3

$S$	$\min \chi^2$
7 [FSS]	140
8 [FSV]	3100
9 [FVS]	4200
10 [FVV]	290
11 [SFF]	3700

“Data” A, using both  $\hat{m}_{\ell\ell}$  and  $\hat{m}_{j\ell}$  distributions:

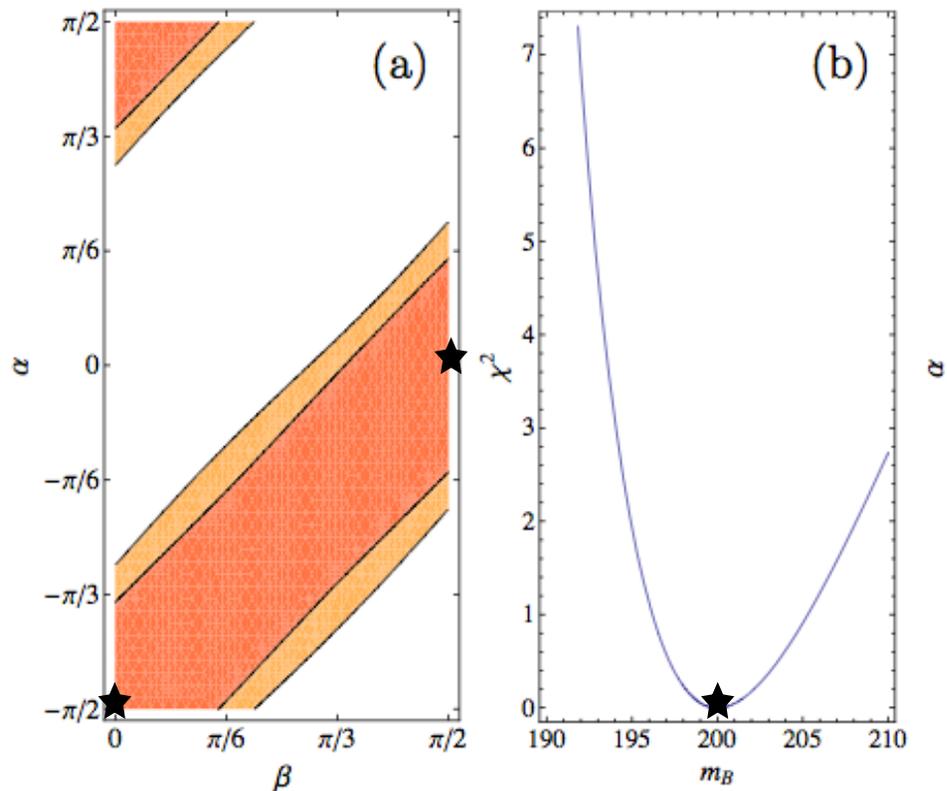
$S$	$\min \chi^2$	best-fit parameters			
		$\alpha$	$\beta$	$\tilde{\gamma}$	$m_B$ [GeV]
1 [SFSF]	0	0.00	1.57	0.00	200.0
2 [FSFS]	150	-0.08	0.07	1.57	754
3 [FSFV]	87	$\pm 1.57$	1.57	0.29	210
4 [FVFS]	48	$\pm 1.19$	0.00	1.57	220
5 [FVFV]	46	-0.93	0.25	1.57	224
6 [SFVF]	37	-0.50	0.53	1.57	197.4

$S$	$\min \chi^2$	best-fit $\tilde{\gamma}$
7 [FSS]	200	?
8 [FSV]	3100	?
9 [FVS]	4300	0.39
10 [FVV]	330	1.57
11 [SFF]	3700	1.08

? indicates that  $\chi^2$  is independent of that parameter

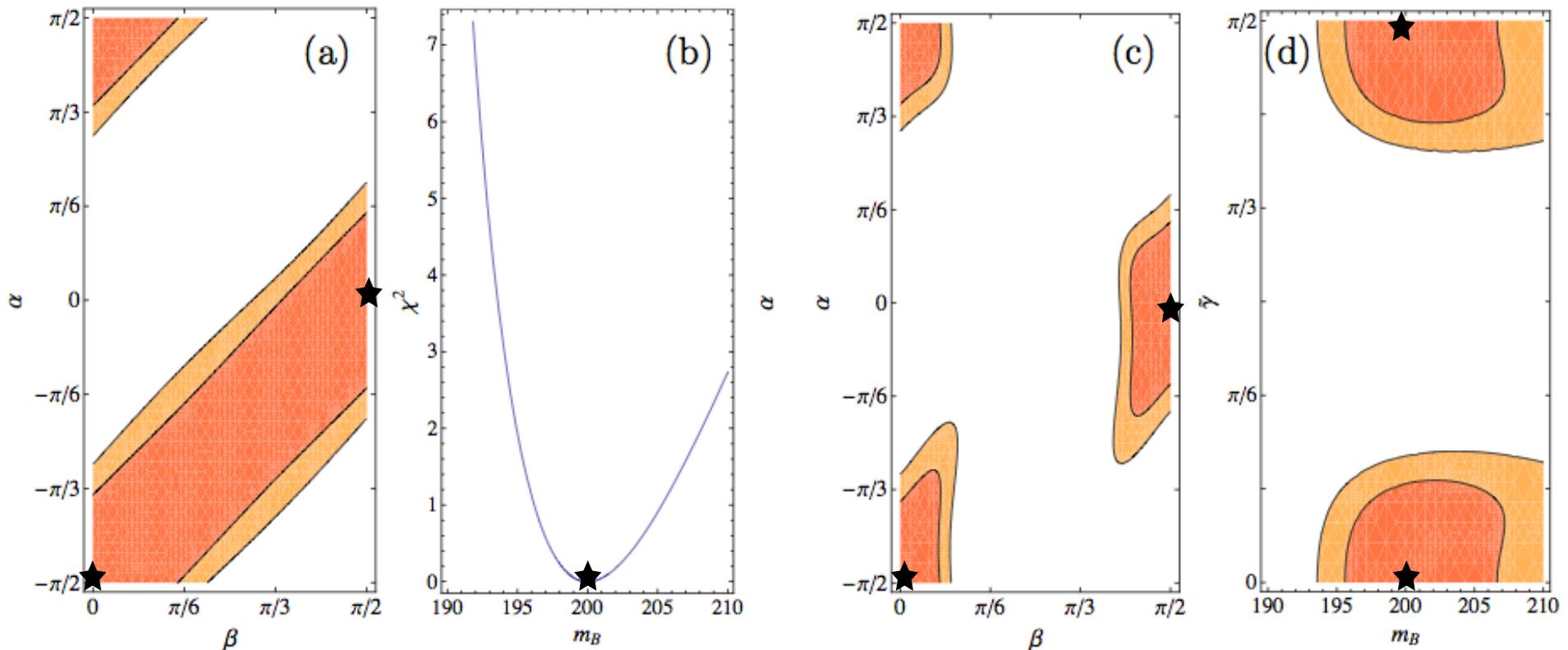
- All possible spin configurations can be discriminated with at least six standard deviations, for a given number of 1000 events.

# Results



- Depict the constraints on  $\alpha$ ,  $\beta$  and  $m_B$  obtained from fitting the  $m^{\wedge}$  II distribution alone.
- The dark/light bands corresponds to 68%/95% confidence-level region.

# Results



- Depict the constraints on  $\alpha$ ,  $\beta$  and  $m_B$  obtained from fitting the  $m^{\parallel}$  distribution alone.
- The inclusion of both  $m^{\parallel}$  and  $m^{\perp}$ , lead to a constraint on  $\tilde{\gamma}$
- Bounds on  $\alpha$  and  $\beta$  are improved.
- The dark/light bands corresponds to 68%/95% confidence-level region.

# Conclusion

- No assumptions about the masses, spins and couplings of new particles have been made.
- Di-lepton invariant-mass distribution is sometimes not sufficient.
- Possible to unambiguously discriminate between all possible spin configuration with high significance.
- The results are based on parton-level analysis. The model discrimination and the precision for the parameter determination may be diluted by jet energy smearing and combinatorics in the realistic experiment but the main conclusions are not affected much.

**Thank you**